FROM MINKOWSKI TO ALEXANDROV

THE PRESENT IN CONVEX SETS

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Imagine yourself invited into a dialogue to discuss some of the most important unresolved issues facing contemporary philosophy of science, including references back to such august figures as Alfred North Whitehead and Gottfried Leibniz — and occasionally Bertrand Russell. You have the chance to listen in and make notes as very large ideas float by in an engaging haze. Such is the dialogue *The Vicarage Iconoclast: Whitehead, Leibniz, Relativity and the Quantum* by Richard T. W. Arthur (henceforth RA). Rather than peppering the text with opaque formulæ we have mostly plain words in a decipherable regular syntax. A splendid achievement. To add a personal note, I was particularly pleased that there is mention of two figures that I had encountered while writing my doctoral dissertation in the early 1980s, and not heard much of since: Arthur A. Robb and Alexandr Danilovich Alexandrov. I had encountered the latter in a short 1972 book by Roger Penrose called *Techniques of Differential Topology in Relativity*. This became the book that I relied on more than any other to deal with issues in my dissertation on structures in space-time. But though the book referred to a 1959 paper (in Russian) by Alexandrov, it contained no information on the man himself, and no indication of what his broader concerns might be. (I suspect that Penrose had been handed this Alexandrov paper by family friend Max Newman — I've often wanted to ask him about it, and, of course, so many other things). It was a great surprise, therefore, to find his topological construction playing a quite central role in RA's dialogue.[1]

Let me note to begin, that the dialogue is ambitious in that it covers aspects of relativity theory as well as quantum mechanics. However, there is less of a sense of multiple disputants having their say on quantum mechanics (henceforth QM) and this, combined with the fact that RA seems to have less investment in this area (as measured by past publications, including a book), makes me also want to spend less

[1] These matters were the subject of my first, and very much unread, publication in philosophy: 'Zeeman-Göbel Topologies' in BJPS for 1988.

time on these issues. So what I have to say on QM will be brief, and glosses over many points of agreement.

The primary focus in this QM discussion (roughly from page 20 to the end) is the relation between Whitehead's process philosophy and quantum theory as understood by Michael Epperson and Ruth Kastner. The most important point that comes from the dialogue is that the wave function encodes real potentialities as opposed to a plurality of real actualities. These potentiae are then mirrors of Whitehead's potentialities. These potentialities are meant to be more than mere possibilities but not actualities. The convex set that represents the mixed states and pure states seems very aptly designed for just this. According to the text, Deleuze is meant to have come up with the idea of a non-metric space consisting of intensive magnitudes, a view that Deleuze apparently took from Leibniz; an intriguing idea but there is no reference given. (One of the very few significant reference omissions in the text.)

Let me add an argument that is not in the dialogue. Whitehead supposedly did not believe in a universe of *things* with their accidents — instead he wanted to insist on processes. Eddington early on expressed the same view. Russell was convinced by at least the first half of this view as well. This accords well with the received idea in QM that bosons and fermions are *indistinguishable* within their kinds and that the former type of particles are associated with the symmetric subspace of a tensor product and the latter with the antisymmetric subspace. (This antisymmetric subspace then gives rise, by virtue of its structure alone, to the Pauli Exclusion Principle.) Of course the idea that quantum particles are not *things* — not tiny billiard balls, in Weyl's words — is strongly suggestive of Whitehead's view and needs to be mentioned. The absence of accidents is then in accord with the no-hidden-variable proofs from Bell to Kochen-Specker, which is in accord with the interpretation of quantum probabilities as potentialities (Incidentally, Henry Krips needs to be added to the names of those who have advocated such a view). This then takes us to one of the main problems of QM: the *measurement problem*. How does measurement select one potentiality from the many and make it actual? In particular, how does measuring one "particle" convey that selection to the other "particle", or "particles", that are entangled with it?[2] This problem is not addressed in the text (whereas it should be, on p. 24) and RA seems not to have taken it seriously enough. Thus with respect to measurement we merely have invocations of decoherence. All fine and dandy, but the noise that might affect a system in a local way is not the same as a measurement that might measure a composite system with nonlocal consequences. Schrödinger gave papers in Cambridge in the early 1930s. Did Whitehead or Russell hear these? Whether

[2]Why are we resorting again to the idea that the particles are distinguishable entities when we have powerful reasons ton beloieve that they are not. Ditto for the application of decoherence. It is a retrograde step.

Whitehead has said anything that adds clarity here is difficult to tell from the dialogue.

Interestingly, when Schrödinger began to become very interested in the whole notion of indistinguishability in the late 1940s he tracked Russell down to Llandudno in Wales, where Russell was holidaying, to specifically ask his views on idea that the individuality of particles may be a chimera. He appears to have come away convinced.

RA speaks of decoherence as *required* for the producing of mixed states. He says: 'given that the effective transition to mixed states presupposes decoherence, the production of actual quantum phenomena, it would seem, requires open systems in order for phenomena to be produced' p. 24. I don't think this can be true. Creating anti-correlated particles creates a composite system that is in an entangled pure state: the component systems are mixed. Decoherence is not 'presupposed' in this. When one of the components is measured the state that was previously mixed now becomes pure. This measurement has created a pure state from a mixed state. So measurement is not decoherence.

I suspect that RA sees measurement of the conversion of a pure state into a mixed state because he holds what has been called the *ignorance interpretation of mixtures*, So when at the end of the measurement the pure state results in a mixed state the latter is nothing more than a pure eigenstate state but-we-know-not-which-one. But there is nothing mysterious – it is all just epistemic. I disagree with this interpretation, but acknowledge it is a hotly contested issue. It seems to me to be another face of hidden variable thinking.

After this RA goes back to the problem of the geometry of space-time. This is the issue that I primarily want to address so I break with these comments on QM to go directly to this, our main concern.

The fundamental question is: does Minkowski space-time admit a concept of the *present* and what structures might subserve it? RA has defended a particular answer to this question over several publications and now in this dialogue. It runs as follows.

Firstly, in Minkowski space-time 'the passage of time, and individual histories, are tracked by *proper time*, not by the time coordinate . . . ' and 'durations are path dependent' (p. 18) This is the standard view and I imagine is now universally accepted. The significant point comes next:

> Savitt[3] and I were each independently taking off from some remarks made by Howard Stein, to the effect that in a fraction of a second all physical interactions between a perceiver and objects perceived can go back and forth through extremely long distances. The perception of what

[3]Steven Savitt is an author of several articles who only a few years after RA, in the same country (Canada), put forward almost exactly the same thesis concerning the present in Minkowski space-time, inspired by exactly the same paragraph in a paper by Howard Stein. Miracles are real, at least in Canada.

> is present would be created during this time, giving you something like the 'presentational immediacy' that is so important to Whitehead. So if you take a fraction of a second of the proper time on the worldline of a putative observer, there will be a spacetime region enclosed between the backward light cone of the end of that second and the forward light of its beginning — a causal diamond,[4] or Alexandrov interval, in the parlance — in which all such interactions must take place. Such a region is defined by us to be the present, not just for the observer, but for any event in the same location in spacetime. p. 18

One preliminary point needs to be gotten out of the way. Alexandrov used these doubled cones (or convex sets, or diamonds, if you must) in order to define a basis for the topology of Minkowski space-time that is anchored in only the physically significant structures — it was used in just this way in Penrose's book — and in which the space-like regions have no significance, and of which we can have no experience.[5] The group that preserves the light cone structure is then precisely the Lorentz group. Or to put it another way: the existence of causal paths — timelike and null — is sufficient to restrict the invariance group to the Lorentz group.[6] The purpose was *not* to define a "present". (I searched through several of the relevant papers by both RA and Savitt and I could find no citation for Alexandrov at all — though I can't claim to have been completely thorough and may have missed something. I'm left wondering where this idea came from.[After writing the previous words I did find an article by Alexandrov — in English! — in the *Canadian Journal of Mathematics* for 1967: 'A Contribution to Chronogeometry.' Could this have been the spark?][7]

But just to emphasise the point so it cannot be missed: it was no part of Alexandrov's intention to use the double cones to define a "present". It was no part of Penrose's intention to do this either. This is because they can't be so used.

There is a historical fittingness about the "double cones": as noted they are convex sets and the person most responsible for developing the theory of convex sets, and convex spaces in general, was none other than Minkowski. Indeed this was his major

[4] These are notable for *not* being diamond-shaped. They are convex sets.

[5] Pimenov and others call this the Interval topology.

[6] Alexandrov's work appeared only in Russian, with no English translations at the time. However his 1949 ideas were rediscovered by E. C. Zeeman in a 1964 paper entitled 'Causality Implies the Lorentz Group' and another in 1967 called 'The Topology of Minkowski Space'. Alexandrov later quipped that Zeeman did not "notice" his own prior discovery of both these topics.

[7] In his 2015 response to Dorato Savitt cites a paper by Minguzzi & Sánchez. from 2008. This paper continues the classification project of the 1980s and so *does* note the proper role of the double cones in the Alexandrov topology, but still cites no work of Alexandrov himself. But just as I myself did in 1980, working on the same project, the authors seem to have relied on Penrose's little book — and this *is* cited by them.

work — and it plays a central role in QM, as the space of mixed states! See his *Theorie der konvexen Körper*. Minkowski is another who is not usually given the credit that he deserves. (Rather confusingly these convex spaces are also called Minkowski spaces — and the buyer must beware which mathematical theory they are getting!) It will come as no surprise that the theory of convex sets was also a major area of research for Alexandrov.

Let us leave for the moment these notes on credit given and not given and return to the philosophical arguments. The claim is that: 'So if you take a fraction of a second of the proper time on the worldline of a putative observer, there will be a spacetime region enclosed between the backward light cone of the end of that second and the forward light of its beginning — a causal diamond . . . ' This is the "present". I think this is all flatly false. I agree that in a split second there will be a great deal impinging on an observer, in the form of sounds travelling on timelike paths, and sights travelling on light-like, or null, paths. And when the sources are close to you the mind is able to blend such experiences seamlessly together — so that the person who is speaking can be seen to be saying the very words that are heard. But this is a physiological matter, rather than metaphysical. The lightning comes to us before, and often long before, the thunder. We do not take these as indiscernibly both present: by the time the thunder comes the lighting is clearly in the past. If our senses were very much more acute than they are then the situation with the lightning and thunder could be more common: we would see the person speaking before hearing their words. Since other animals have much more acute senses than we do — I'm thinking of birds, in particular, who can hear things at a very great distance — then this may be something they experience more regularly. These physiological limitations have a social benefit: they bond us together in what seems to us a common sensorial present, while what is distant breaks apart into separate sights and sounds.

The second thing is that a double cone does not play any role in this; it is irrelevant. Take two points on a timelike path lasting a spit-second and label the earlier point A and the later point B. Now take the intersection of the future cone at A with the past cone at B. This is the *double cone* for these two points. No points in this dbl cone are *the present* of the point A — since they all lie in its future. *This dispenses with any such earlier end point* A. If we then eliminate successive end points with the interval path getting shorter then we will eventually reach just the single point B. So now look at point B: what is present to it? Well, all of the points in the double cone are in its past — and if points in the past are to be allowed to count then there are many points in its past but not part of the double cone that should count as well. So for the points A and B the double cone does not define a present. Now take a point on the timelike path between A and B but different from both. Call it C. Now we simply combine the previous two arguments to see that points in the future of C cannot count at all, while

if points in the past of C are to count as the present then there are many points in the past that have not been included and we have been given no principled reason to exclude them. If the response to this is to note the points that are space-like separated from C but still within the double cone then these points have no experiential meaning for us at all, they are not "simultaneous" nor "contemporaneous": their inclusion in the double cone is *de trop*. And we need not even mention that the widest point of the dbl cone is wide with lots of these irrelevant space-like separated points when C is near the midpoint between A and B. So I repeat: this view is flatly false.

I think RA and Savitt are, in espousing their view requiring the introduction of the double cones of Alexandrov, being seduced by the *Fallacy of the Fancy Commonplace*. I feel allowed to christen this fallacy in this way as RA has blazed the trail here, and has named and deployed quite a few new Fallacies of his own — such as the Fallacy of Misplaced Concreteness (p. 8). In fact the Fallacy of the Misplaced Concreteness might actually be a super-fallacy category that encompasses my own meagre contribution. Here is what I mean by the fallacy of the Fancy Commonplace: there are certain concepts that are part of our ordinary world-view but that physics does not support. For example we want the microphysical world to contain *things* with their accidental properties. We feel that if we do not have this then a philosophical catastrophe has beset us: how will we name and characterise? how will we quantify and formalise? how will we speak of things truthfully? what about Leibniz? Philosophers are especially sensitive to the crisis that this creates; we straddle two realms: of the common-sense macroscopic world and the mathematical constructions of theoretical physics, and we want, and need, these two things to mesh with another — and frankly we are the only ones who are professionally concerned with this reconciliation. We get a victory if we can make use of the mathematical and physical constructions but keep our commonplace ordinary world-view from crumbling. So it is also with the subject here: the present is important to our commonplace worldview of time and so it is seductive to invoke some little known structure of Minkowski space-time in order that the reconciliation be effected. The commonplace = the present, the fancy = the double cones (or better, since fancier, the "diamonds") of the topology. It hardly matters that it makes no sense at all.

It should also be noted that it would not be possible to add any *additional* structure to the null cone set (I intend here to include the timelike curves of the interior), for we already know that this structure implies invariance under the Lorentz group. Indeed to repeat ourselves: this was proven by Alexandrov. This all needs to be emphasised since sometimes Savitt seems to want an *addition* because Einstein implied that it is required. But anything additional — i.e. not entailed by the existing light cone structure — would change the invariance group.

The claim that the double cones cannot play the role they've been assigned here

has been made before, by Mauro Dorato and there is a response by Savitt (which is distinguished by the fact that it may be the first paper to have emojis for a title). However before defending Dorato I want to say that I think his expression, following the earlier Savitt paper, for the RA and Savitt view as 'the Alexandroff present' is particularly egregious — and it is so much worse to then abbreviate this to "Alex" (which, to remind the reader, was Alexandrov's first name). What is Savitt's response, particularly to Dorato?

Dorato expresses his rejection of the RA and Savitt view rather broadly. It is 'neither a physically relevant property nor a very good *explanans* of our temporal experience. Therefore, it should be dropped.' I have argued that the first part of this is wrong: the topology of the space, derived as it is from the null cone structure, could hardly be said to be physically irrelevant. However Dorato fully acknowledges this later in his §4. And Savitt has an easy time pointing to its uses. The second point, however, is correct: It is not 'a very good *explanans* of our temporal experience'. I've already argued this point but to conclude I see nothing more that is needed to explain our temporal experience than Minkowski space-time. It does not seem to me that Savitt makes any effort to defend the proposal but rather concedes everything that is important to Dorato. Savitt says (Savitt p. 21)

> As I point out (352), but as we all knew already, in the special theory of relativity there is no such distinguished set of simultaneous events. So Dorato is surely right when he says that causal diamonds, if proposed as a scientific successor concept to our common sense concept of the present, do "not correctly pick out the events we intend to pick out when we use 'now' in ordinary language." It is true, however, that *nothing* in *M* does.

Agree completely! How has Dorato been in any way refuted? But if we are back to a physiological explanation then we might just leave it there. We are not really doing metaphysics, much less philosophy of science. But Savitt wants to suggest that we *are* still doing philosophy — or at least, we *perhaps* are!

> Alternatively, if one wishes to see what elements of our pre-relativistic concept of time one can find in relativistic spacetimes, one can seek some elements of or structures in Minkowski spacetime (or the more general class of spacetimes stipulated earlier) that *more-or-less* play the role that the commonsense present did. If one does make such a proposal, one knows in advance that it will not encompass precisely the set of points intended when we use "now" in ordinary language. One looks for a "best fit," with the criteria of fitness rather loosely specified. That is the philo-

> sophical task — assuming that there is a philosophical enterprise here at all. (p. 21)

This idea, that we must look for 'some elements of or structures in Minkowski space-time' that will 'play the role that the commonsense present did' is a form of gibberish common in much of modern philosophy. The structures or elements of Minkowski space-time are such elements of affine geometry as points, vectors, and metrical relations inducing topological features — these cannot "*play the role*" of something denoted speciously as a "commonsense present". Such nonsense is compounded by then trying to make this "playing the role" be a "more-or-less" matter.[8]

Dorato was correct, I think. To explain an aspect of our experience it is necessary to invoke the physiological processes and limitations that are responsible for that experience. This means acknowledging that brain processes are not instantaneous, they take time; perceptual discrimination has its limits; and human bodies are at the apexes of uncountable light cones.

I would say — and did once say, in an unpublished paper[9] — that the feeling that time is flowing is not due to a movement of time but rather that there is something that is changing that we are imagining as unchanging: this is the *Self*, or the *I*, or the *ego*. We falsely imagine this Self as a static element with change in the things about us and us observing that change. We imagine something as flowing relative to we who are static but in reality we are changing in just the same way as the world is changing. The illusion of something flowing is caused by the illusion that something that is flowing isn't. So the idea of ourselves as being in a present is part of the same illusion: the present becomes the moment we have trapped the Self in.

What we take to be the present is best broken into two parts: a *visual present* and a *timelike present*. The visual present at time t is simply the sheath of the past light cone of t as it intersects t. It is a two dimensional non-Euclidean sphere centred on a point at some moment of time: this is the absolute sky. There is also, relative to the observer's frame of reference, a relative sky: Penrose considers it to be a Riemann sphere CP^1. This has a conformal structure, but no intrinsic metric. In the process of making a change in reference frame the angle between curves on this sphere is preserved, this giving rise to a Lorentz-Fitzgerald contraction from one observer relative to another. In this relative sky star positions may change but, owing to the preservation of angles, the shapes do not. Thus circles are mapped to circles. But what is seen by us is still a function of what light is reaching the observer: each light ray that reaches the observer's eye corresponds to a single point on the observer's sky. It is not significant

[8] Coming soon, the new film: Philosophy 101: The integers play the role of sadness.

[9] Tentative title was 'The Heraclitean Myth'; this was written long ago, in 1990. It was conceived as a comment on the famous query of Jack Smart's about the flow of time (How fast would it be flowing? At one second per second?) while I was housesitting for him in Canberra.

how far away the source of the light might be. The surface of the sphere being two (real) dimensional it is spread about us in a space of three dimensions. This is what we take to be the *space* of our visual spatial experience. But, it is important to emphasise, our spatial experience is not based in any way on what is space-like separated from the observer's standpoint. The term "space-like" was perhaps poorly chosen, in that it inevitably seemed to suggest the contrary.)

This then takes us to the *timelike present*. It is much more difficult to characterise and I don't think we have a very clear sense of it: the visual present dominates. But if we are being rained upon, walking in the wind, or we are engaged in a conversation (of the old-fashioned kind, with the interlocutors actually present) then the fact that the sound travels slower than the speed of light suggests to us that these are part of the timelike present. In addition there are things that travel between the speed of sound and the speed of light and such things may give no warning: a bullet or a meteor, for example. These things are, when the reach you, also part of your timelike present. None of this should surprise us in any way. But such incidents, I would guess, register on us far more rarely than the visual now. We are swamped by the visual. These two presents, visual and temporal, merge together — as in the afore-mentioned example of two people talking together: one sees the other speaking and hears the words with no discernible delay. The space of timelike vectors reaching a point t is three dimensional and its surface is wrapped by the two-dimensional sky of the null cone. We tend to identify the timelike presence with the slower than light influences that can touch us. The hitch comes when we consider what this I is. Or equivalently, present to what? If we are going to consider a point and the backward light cone of a point as determining the present then we have to ask what it is, or who it is, that is at this point. But of course we don't exist at a point just as we don't have light solely impacting our the part of our nervous system restricted to our eyes. One of the functions of our skin is also to receive and make use of light. And our nervous system is spread throughout our bodies making use of all stimuli. There is no single point in our bodies that can be identified as the single point-like apex of one light-cone. Rather there are many such points and integrating them is a task that is not localisable to any one site in our bodies. We are not one worldline, we are an immensely large bundle of worldlines in interaction with one another along their own light-like and timelike paths. This point has been made before, sketched by Schrödinger and then made more pointedly by Stein. In fact this may well be the very paragraph that is supposed to have ignited the same idea in RA and then Savitt.

> Now, in the theory of relativity, the only reasonable notion of present "*to a space-time point*" is that of the mere identity-relation: present to a given point is that point alone — *literally* "here-now" (see Stein 1970b,

> 15). On the other hand, the set of events contemporaneous with a specious present will always be a spatially extended one. And it is, I think, of very great relevance to the misconception I am trying to dispel, that this spatial extent — although finite —is in fact *and in principle, as a matter of physics*, always, in a certain sense, immensely large. I must explain in what sense "large", and on what basis in physical principle. Stein p. 159

The idea is that the ratio of the largeness of our bodies to the much greater distances that light can travel is supposed to give us a comparatively large specious present. He says:

> To say (in this connection) that the velocity of light is "great" must be understood in the sense, not that this velocity is expressed by a large number, but that during a specious present, light travels a spatial distance that bears a very large ratio to the spatial extent of our bodies or of ordinary objects.

Note that this does not define, or attempt to define, a specious present. It associates the ratio of sizes to whatever we might think is the specious present. But we have again the physiological answer to the question of why there is a sense of a short period of time in which light-like and timelike processes are sufficiently granulated and treated as simultaneous processes. the specious present is due, in other words, to the same processing issue, that gives rise to the illusion of time flowing about us: we take ourselves as a constant (a Self) over a short time period, not realising that we are changing as everything about us is; so also that constancy gives us a specious moment in which that self has a world present to it. The two illusions are linked together, by our physiological limitations.

I want to register a disagreement with Stein on this. Stein stresses the large spatial distance that light can go in a small amount of time. But I'm not sure this matters a great deal, just as it doesn't much matter that we are receiving light that was emitted millions of years ago. What matters is how fast light goes, not how far it has come from. The processes in the body are slowed down to the speed of electro-chemical and chemical interactions — in other words to travel on timelke paths — and this is what matters most for our sense of a present. A creature that had more efficient sensory processing would have a different sense of a specious present and possibly none at all. So let us put up a thought experiment to test Stein's intuition: how would our experience change were light to travel at half its current speed, but all the other principles of relativity remain as they are (so the speed of light is constant in all

reference frames, that it is a maximum for timelike processes, etc). Would we notice any difference at all? I conjecture that we would not.

But the physiological argument that Stein is giving runs counter to the guiding idea of RA and Savitt that the convex sets of the double cones are significant structures for our sense of a specious present. For these have no natural scale at all: some may be very large and others very, very small. The size of them would be a *metrical* fact but they aren't designed to have any metric presuppositions at all, they are picked out for the purposes of defining a topology — and are thus pre-metrical. And, as I argued earlier, at whatever size they are they cannot accomplish what they are supposed to accomplish: almost all of the points in them are completely irrelevant. The relevant points are the intersection of the past light cones with the observer's world-points.

The physiological argument above is a form of subjectivism, which brings us back to Whitehead — of whom Stein was an admirer, according to his obituary on Stack Exchange. Would Whitehead have agreed with this argument. Perhaps, but it is unlikely that would have thought that it went all the way to the metaphysics he wanted. Perhaps one should just say: so much the worse for Whitehead's metaphysics. He is not really interpreting relativity, he is resisting it. He wants a world where there is a genuine classical present and he would like that to be overlaid onto relativity theory. And, frankly, that seems to be what RA and Savitt are also doing with their specious idea of a specious 'present'.

Nevertheless we have RA to thank for making the sense that he has made from Whitehead's remarks. It cannot have been easy. It would be good to see more; but the next time with an attempt to perhaps reduce it to a set of axioms or postulates. It is extraordinary that the man who wrote *A Treatise on Universal Algebra* did not attempt to do that himself.

I haven't yet touched on the whole issue of Internal vs External Relations — which plays a very large part in the dialogue, and tied Whitehead together with Leibniz, and must separate both from Russell. Let me just note briefly that the existence of entanglement seems to me to definitively rule against the view that all relations are internal relations. But the existence of multipartite entanglement seems to rule against an all-or-nothing relation suggests that it must be an *intensive relation* in order to account for *entanglement monogamy*. I don't see anyone proposing this in the literature (though of course I may have missed something). Nevertheless this puts us quite some way beyond simplistic discussions of whether external relations exist and into which Russell once again turns out to be on the side of the angels. But entanglement monogamy also interposes some moderation into Whitehead's universal holism (which he borrowed from Hegel?) — everything is connected to everything else — as it makes it clear that entanglement between two parties must be at the expense of entanglement with any third party. Thus we need to see holism as having grades and

nuances of its own: it is not one view but splits into many. The holism of Nicholas of Cusa is not the same as that of Hegel and this in turn is not what we find in QM. Indeed if the holism of Hegel were to reign then it is not clear that physics would be possible at all.

It is worth adding to this that if any form of holism stands the test of experience and experiment then it constitutes an insurmountable obstacle to modal logic under its standard possible worlds semantics. If A and B are connected together so that they will vary together then this must be a constraint on what configurations are possible and so on which possible worlds can exist. Hume was famous for saying that 'All things seem entirely loose and separate' but one must be careful to put proper emphasis on that '*seem*'. If the evidence were that Hegel's, and Whitehead's, holism were correct then that would mean that there is only one possible world: the *actual* world. So also, in a far more complicated way — and probably forever beyond our computational reach — would it be the case that a holistic QM makes some other possible world than the actual world very unlikely. And into this there also enters the problem of indistinguishable particles which will also impact upon the problem of what is a possible alternative to a given situation.[10] It has only been by a shear irrational pretence that modal logic has been able to be held up as having an application to the actual world we live in. We either need to give it a new semantics or discard it as an unhelpful fantasy.

Problems with modal logic will overflow, of course, into broader philosophical issues, particularly into the counterfactual analysis of, or criteria of, causal connections. What has been proposed has not worked even in classical scenarios much less anything quantum mechanical. It is too easy to overlook the fact that causality has not arisen naturally out of QM in the same way that entanglement has. Quantum field theory may be a different story.[11] But even in the case of quantum field theory there is *still* no mathematically rigorous theory of interaction for massive particles in four dimensions. There seems to be an increasing scepticism that it will ever be found.

This in turn puts Minkowski space-time in the cross-hairs, since the reconstruction of the 1950s–60s made plausible the idea that causation was the fundamental relation on which Minkowski space-time was based. This might be called the thesis of the Leningrad School, of which the leader was, undoubtedly, A. D. Alexandrov, but included Pimenov, Tal'chik, Ovchinnikova, Vallander, and others — with major influence from the German Herbert Busemann.[12] This then passed to Penrose in

[10] Swapping A and B may be an impossibility if they are not distinguishable in the first place.

[11] See Heathcote 'A Theory of Causality: Causality = Interaction (as defined by a suitable Quantum Field Theory)' *Erkenntnis*, 1989, 77–108

[12] A. A. Robb's work also seems to have been a fundamental inspiration here. But Alexandrov was a staunch Leninist and his ideas may have carried this ideological disposition also from the beginning.

the mid 1960s and became the project to put causality constraints on space-times as a way of ruling some out as being pathological. Hawking and Ellis' famous book *The Large-Scale Structure of Space-Time* is part of that same project. But the more amorphous question of *what this causal influence is* was left unanswered — a blank to be filled in later. Penrose had the idea quite early on to shift attention to the *spinor structure* — since this at least had a clear quantum mechanical significance and also, just as significantly, a geometric significance. This then became the basis of his *Twistor* theory.

It is not difficult to see that we have here a house of cards. One possibility is that we are just wrong in thinking of space as being one single structure, or of having only one structure on it: perhaps it breaks into two parts: a commuting space and an anticommuting space — the former for bosons and the latter for fermions. But then we still have to find a way to accomodate relativity theory and entanglement; thus we should still need a Minkowski kinematical structure, but perhaps also a measure structure to accomodate the latter(?). So perhaps what we need is not a home for causality *per se* but a space that has a temporal structure: so a *timelike space* (Busemann's phrase).[13] And what about all the fundamental forces? What would that look like? Of course I have less than no idea. But then metaphysics is currently little more than a cluster of loose ideas that mask unacknowledged geometries which are in naked conflict with one another.[14] If we imagine this as a dialogue then it would be far from harmonious.

[13] This then would rely on Nachbin's ordered topological spaces. Also fundamental and required for the Alexandrov topology.

[14] If there is anything notable about Whitehead's ideas it is that they do *not* mask algebra or geometry; they mask a spiritualism!